

# Reducing Standing Waves in Quasi-Optical Systems by Optimal Feedhorn Design

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**Abstract**— Standing waves between transmit and receive feedhorns in quasi-optical systems often limit the achievable performance of mm-wave and sub-mm-wave instrumentation. Even with high performance corrugated feedhorns and perfect frequency independent optics, significant standing waves can occur because of the resonant build-up of higher order modes between feedhorns. In this paper we describe a new design of wideband corrugated feedhorn that significantly reduces standing wave effects, is scalable to any frequency, is shorter than standard horns and is suitable for a wide range of optical configurations. In addition it produces far-field beam patterns with much reduced sidelobes. We will describe the theory behind this new feedhorn design, outline scaling laws and present experimental results confirming the analysis.

**Index Terms**— Antennas, feedhorn, gaussian beams, millimeter wave, quasi-optics

## I. INTRODUCTION

IN the mm-wave and sub-mm-wave regime increasing losses in single-mode waveguide often lead to the choice of quasi-optical instrumentation for the highest levels of performance, with single-mode components coupled together using corrugated feedhorns and Gaussian optics. In principle, these systems offer low loss transmission and work over relatively large bandwidths [1]. However, in some cases, large standing waves are observed in apparently very well-matched systems. with such effects usually associated with modal resonances. In this paper we present a corrugated feedhorn design that significantly reduces their effect, whilst offering far field beam patterns with very low side-lobes and phase centres at the horn aperture.

## II. DESIGN PRINCIPLES

### A. Modal Resonances

Modal resonances can occur in any system where two single mode networks are coupled together via an overmoded transmission line. Even a small scattering to a higher order mode in the transmission line can lead to that mode becoming resonantly trapped between the two single mode networks and power building up in that mode. In a completely lossless system, scattering back to the main transmission mode, with the appropriate phase, can even lead to zero transmission of the main propagating mode at certain frequencies. In practice,

the magnitude of the effect depends on the relative size of the scattering to the higher mode compared to the losses of the higher order mode in the system.

Approximate analytical expressions can be calculated using a scattering matrix approach first described in detail in 1956 [2]. That analysis indicates that modal resonances can be eliminated by careful design of the transmission system.

### B. Quasi-Optical System

A generic quasi-optical system consists of two identical corrugated feedhorns coupled together by focusing Gaussian beam optics. The horns are usually designed to couple from a  $TE_{10}$  single mode rectangular waveguide via the  $HE_{11}$  mode in the corrugated horn to the fundamental Gaussian mode at the aperture. In practice,  $HE_{1n}$  modes are also often excited in the horn and higher order Gaussian modes are often excited at the aperture, both with small but significant amplitudes.

It can be shown that modal resonances can be eliminated if all the significant  $HE_{1n}$  modes excited in the feed arrive exactly in phase or out of phase with the dominant  $HE_{11}$  mode at the aperture. The worst case is when a  $HE_{1n}$  mode arrives 90 degrees out of phase with the  $HE_{11}$  mode which is likely to minimize the coupling and maximize the standing wave effects. The Gaussian optics should also recreate the same amplitude and field distribution at the input of the receive horn that appeared at the output of the transmit horn. In other words any higher order Gaussian modes excited at the aperture of the transmit horn should arrive in phase with the dominant fundamental Gaussian mode at the aperture of the receive horn (for horns with phase centers at their apertures). This can be achieved using zero gain frequency independent optics, where the focusing elements all have the same focal length and are separated by the sum of their focal lengths.

In principal, the analysis is simplified if only the  $HE_{11}$  mode is excited within the horn. Alternatively, the optical design is simplified if only the fundamental Gaussian mode is excited at the aperture of the horn.

### C. Maximising Coupling to Fundamental Gaussian Mode

In many previous analyses of corrugated feedhorns and quasi-optical systems it was assumed that the primary design aim was to excite only the  $HE_{11}$  mode, which then couples with ~98% efficiency to the fundamental Gaussian mode. Simple fundamental Gaussian beam propagation is then used to design the optics. However, although only 2% of the power is contained in higher order Gaussian modes, it is these modes that can contribute to modal resonances (especially with inappropriate optics), and lead to sidelobes in the far field antenna pattern. These problems would be significantly reduced if the horn could excite a much purer fundamental Gaussian mode. Figure 1 indicates the efficiency with which

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combinations of  $HE_{1n}$  modes are excited by a pure fundamental Gaussian beam focused to a beam waist  $w$  at the aperture of a corrugated feed for an aperture radius  $a$ . By extension, if these modes could be excited in the corrugated feed with the correct amplitude and phase distribution then they would excite a much purer Gaussian mode. Figure 1 shows that a simple combination of  $HE_{11}$  and  $HE_{12}$  modes with the correct amplitude and phase could excite a beam with 99.8% purity for  $w/a \sim 0.5$ . An appropriate coupling analysis shows that the  $HE_{12}$  mode should have a relative amplitude of 0.3 (9% of the total power) and should be in phase with the  $HE_{11}$  mode ( $\sim 91\%$  of power).

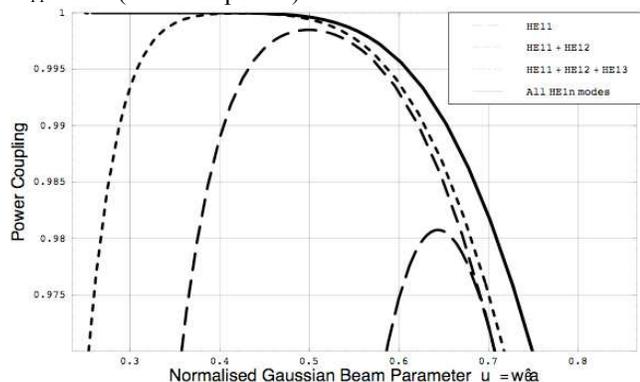


Fig. 1. The relative amounts of  $HE_{1n}$  modes excited at the circular aperture (of radius  $a$ ) of a corrugated feedhorn by a linearly polarized fundamental Gaussian mode with a beam waist  $w$ . Note for large  $w$  a significant amount of power is outside the horn aperture, limiting efficiency. Note that by extension a linear combination of  $HE_{11}$  and  $HE_{12}$  modes (in phase) can excite a fundamental Gaussian mode approaching 99.8% efficiency.

#### D. Horn Design

The basic design is shown in Figure 2. It is well known that profiled horns can excite the  $HE_{12}$  mode in addition to the dominant  $HE_{11}$  mode. Modal analysis showed that for horns where  $a \gg \lambda$ , the  $HE_{12}$  mode can be excited with a relative amplitude of 0.3 for a profiled horn length of  $\sim 2.4a^2/\lambda$  (where  $a$  is the desired horn aperture radius and  $\lambda$  is the wavelength). However, the  $HE_{11}$  mode and  $HE_{12}$  modes are then badly out of phase at the aperture and produce a distorted optical beam pattern. However, the two modes can then be brought back into phase if a straight section of corrugated guide of length  $\sim 2a^2/\lambda$  is added to the profiled horn. The two modes can then combine to produce a highly pure fundamental Gaussian mode with very low sidelobes in the far field.

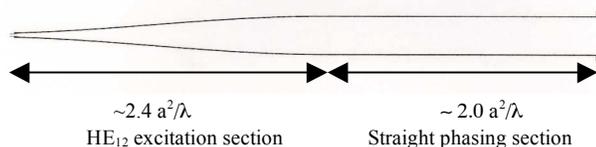


Fig. 2. The corrugated horn profile consisting of a profiled section designed to excite the desired  $HE_{12}$  amplitude followed by a straight corrugated phasing section designed to bring the  $HE_{11}$  and  $HE_{12}$  modes into phase at the aperture.

### III. EXPERIMENTS

These predictions have been experimentally tested at 94GHz using large aperture feedhorns of diameter 46mm. Profiled horns of length 400mm with a radial  $\sin^2$  profile (without the

straight phasing section) gave rise to significant standing wave effects when the horns were coupled using high quality frequency independent optics. In addition, poor far field beam patterns with high side lobes were observed. However, the addition of straight phasing sections of corrugated pipe of length 330mm reduced standing wave effects by more than an order of magnitude (to a level that could be caused by simple mismatch) and significantly increased coupling efficiency. It also led to a reduction of far field sidelobes by more than 15dB to -36dB in good agreement with theory. It should be noted that the overall length of the feedhorn appears large only because of the very large horn aperture. If the length is scaled for aperture size, the total length of the feed is significantly smaller than a standard linear taper feedhorn. It should be noted that the exact dimensions required for the feed were determined using the mode matching software program CORRUG from Antenna Software Ltd.

### IV. DISCUSSION AND SCALING LAWS

This horn design has some similarity to a Potter Horn in that both rely on exciting two modes and bringing them in phase at the aperture. However, unlike Potter horns, this horn design is found to have excellent beam patterns that can extend over large 20% bandwidths. These horns also have the advantage that the phase centre is always exactly at the aperture, largely independent of frequency. There is also a clear design methodology where mode excitation and phasing are clearly separated and a scaling law that allows the system engineer to calculate the required length of horn for a given aperture. For a wide variety of  $w/a$  parameters the required length of horn is found to be  $\sim 4.4a^2/\lambda$ . This can be compared with the phasing length required to achieve a full  $360^\circ$  phase difference between the  $HE_{11}$  mode and  $HE_{12}$  mode in a straight corrugated guide is  $\sim 3.2a^2/\lambda$ .

In practice, modeling shows that virtually all corrugated circular feedhorns excite the  $HE_{12}$  mode to some degree and that the horns performance can nearly always be significantly improved by ensuring that this mode arrives in phase with the  $HE_{11}$  mode.

Fig. 1 shows that, in principle, even higher purity Gaussian beams can be produced by exciting a different mode set that also includes the  $HE_{13}$  mode, although it becomes more difficult to arrange for all three modes to have the correct relative amplitudes and phase relationship, and the horn aperture efficiency becomes significantly reduced.

Further explanation and details will be provided in the talk.

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